Using the product of two mutually perpendicular truncated polynomial series as shape function for rectangular plate analysis

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ABSTRACT
This paper presents deflection function for plate analysis in the form product of two mutually perpendicular truncated polynomial series. The aim herein is to adopt this function as a very good approximate deflection function for first mode analysis (pure bending, stability, vibration and thermal bending) of plate continuum. Energy methods (Galerkin, Ritz, work principle etc) are veritable tools that employ this function for first mode analysis.. When the polynomials are truncated at the fifth term, the kinematic and the kinetic boundary conditions are satisfied. In the same way the obtained data from using this truncated series function in energy methods are very close to the date obtained from numerical and other methods.

1. INTRODUCTION
Until the year 2012, when Ibearugbulem (2012) introduced the use deflection function for plate buckling analysis in the form product of two mutually perpendicular truncated polynomial series (otherwise hereinafter called, “polynomial function” for brevity), most scholars and students of structural engineering were perplexed by the complexity of plate analysis. The reason for the perplexity was explained by Han et al (1989) when they opined that use of trigonometric beam functions in plate analysis induces complexity in integration. This complexity in Integration resulting from using trigonometric beam functions was the main reason numerical methods of plate analysis evolved. However, the numerical methods pose their own problems as sound knowledge of use of computer is inevitable. This problem lead to the evolution and use of orthogonal polynomial function in plate analysis (Malhotra et al, 1987; Kim, 1988; Han et al, 1989; Lewi and lam, 1990; Singh and Saxena, Rizk and Ashour, 2001; Kim et al, 2012). With this orthogonal polynomial function, the perplexity of plate analysis still subsists. This problem explained above is the major crux of this paper. Now the question is: can this polynomial function be use in energy methods for all kinds of plate analysis?

2. OBJECTIVE OF STUDY
The main objective of this study is to employ polynomial function in energy method for all kinds of plate analysis in first mode.
Another objective is to know whether this new approach is cumbersome or complex to handle and to know whether obtained results will be reliable.

POLYNOMIAL FUNCTION FOR PLATE ANALYSIS

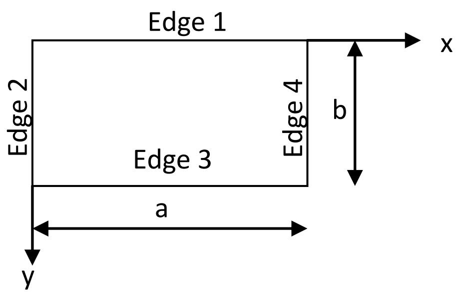

Figure 1: rectangular plate showing edge numbering and dimensions in x and y axes

Ibearugbulem (2012) developed and gave the general deflection equation of plate in polynomial series form as:

$$w = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \alpha_m R^m . \beta_n Q^n \quad (1)$$

Where α and β are coefficients of the series in x and y axes respectively. R and Q are the dimensionless variables in x and y axes respectively and are defined as R =x/a and Q = y/b. m and n are the powers of the terms of the x and y axes series respectively. a and b are the dimensions of the plate in x and y axes respectively. He further truncated the series at the fifth term for both x and y axes series as:

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} \alpha_m R^m . \beta_n Q^n \qquad (2)$$

Expanding equation (2) gives:

$$w = (\alpha_0 + \alpha_1 R + \alpha_2 R^2 + \alpha_3 R^3 + \alpha_4 R^4)\ (\beta_0 + \beta_1 Q + \beta_2 Q^2 + \beta_3 Q^3 + \beta_4 Q^4) \quad (3)$$

In his work, he discovered that truncating the infinite series at the six term for x and y axes series did not improve the obtained result, hence he recommended that it is best to truncate the series at the fifth term. Thus, equation (3) is the polynomial function deflection equation for plate analysis. By satisfying the kinematic and natural (kinetic) boundary conditions for SSSS, CCCC,CSSS, CCSS, CSCS and CCCS plate gave peculiar deflection equations for the plates as shown on table 1:

Table 1: Peculiar polynomial deflection equations for plates with no free edge

PLATE	DEFLECTION EQUATION
SSSS	$w = A(R - 2R^3 + R^4)\ (Q - 2Q^3 + Q^4)$
CCCC	$w = A(R^2 - 2R^3 + R^4)\ (Q^2 - 2Q^3 + Q^4)$
CSSS	$w = A(R - 2R^3 + R^4)\ (1.5Q^2 - 2.5Q^3 + Q^4)$
CCSS	$w = A(R^2 - 2R^3 + R^4)\ (Q^2 - 2Q^3 + Q^4)$
CSCS	$w = A(R - 2R^3 + R^4) . (Q^2 - 2Q^3 + Q^4)$
CCCS	$w = A(1.5R^2 - 2.5R^3 + R^4) . (Q^2 - 2Q^3 + Q^4)$

Similarly, by satisfying the kinematic boundary conditions and the shear force natural boundary (without satisfying bending moment natural boundary) conditions for SSFS, CCFC, SCFS, CSFS, CCFS and SCFC plates gave peculiar deflection equations for the plates as shown on table 2:

Table 2: Peculiar polynomial deflection equations for plates with one free edge

PLATE	DEFLECTION EQUATION
SSFS	$w = A(R - 2R^3 + R^4)\ (8Q - 4Q^3 + Q^4)$
CCFC	$w = A(R^2 - 2R^3 + R^4)\ (4Q^2 - 4Q^3 + Q^4)$
SCFS	$w = A(1.5R^2 - 2.5R^3 + R^4)\ (8Q - 4Q^3 + Q^4)$
CSFS	$w = A(R - 2R^3 + R^4)\ (4Q^2 - 4Q^3 + Q^4)$
CCFS	$w = A(1.5R^2 - 2.5R^3 + R^4)\ (4Q^2 - 4Q^3 + Q^4)$
SCFC	$w = A(R^2 - 2R^3 + R^4)\ (8Q^2 - 4Q^3 + Q^4)$

3. RITZ TOTAL POTENTIAL ENERGY FUNCTIONAL AND EVALUATION PROCEDURE

The Ritz total potential energy functionals for pure bending, buckling and vibration are respectively given as:

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \left(\left[\frac{\partial^2 w}{\partial R^2}\right]^2 + 2\frac{1}{p^2}\left[\frac{\partial^2 w}{\partial R \partial Q}\right]^2 + \frac{1}{p^4}\left[\frac{\partial^2 w}{\partial Q^2}\right]^2\right) dR\, dQ - ab\int_0^a\int_0^b (qw)\, \partial R\ \partial Q \qquad (4)$$

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \left(\left[\frac{\partial^2 w}{\partial R^2}\right]^2 + 2\frac{1}{p^2}\left[\frac{\partial^2 w}{\partial R \partial Q}\right]^2 + \frac{1}{p^4}\left[\frac{\partial^2 w}{\partial Q^2}\right]^2\right) dR\, dQ - \frac{Nx}{2}\int_0^a\int_0^b \left(\frac{\partial w}{\partial x}\right)^2 \partial R\ \partial Q \quad (5)$$

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \left(\left[\frac{\partial^2 w}{\partial R^2}\right]^2 + 2\frac{1}{p^2}\left[\frac{\partial^2 w}{\partial R \partial Q}\right]^2 + \frac{1}{p^4}\left[\frac{\partial^2 w}{\partial Q^2}\right]^2\right) dR\, dQ \ - m . \lambda^2\int_0^a\int_0^b w^2\, \partial R\ \partial Q \quad (6)$$

Where p is the aspect ratio given as: p =b/a

It will be noticed that what is involved in equations (4), (5) and (6) are partial differentiations and partial integrations. A close look at tables 1 and 2 reveals that carrying out partial differentiation of those polynomial functions with respect to R and Q will not pose any difficulty or complexity at all. In the same manner, integrating the resulting functional with respect to R and Q after substituting the partial derivatives of the polynomial functions into equations (4), (5) and (6) will not pose any problem as it will be straight forward. Carrying out partial differentiation of the equations (4), (5) and (6) with respect to A after the integrations will result to an equilibrium function (equation) from where the problem is solved.

4. PRESENTATION OF DATA

Some results obtained from using polynomial functions as deflection equations in energy methods are presented here. These data were compared with those from earlier studies.

Pure bending analysis of SSSS plate

The maximum deflection parameter, w_1 of the plate was determined by Ibearugbulem et al (2013) using the method describe herein and they compared their values with those (w_2) obtained by Timoshenko & Woinowsky-Krieger (1959) as shown on table 3 (note % Diff. means percentage difference).

Table 3: Maximum deflection parameter for SSSS plate

b/a	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
P =a/b	1.0	0.9091	0.8333	0.7692	0.7143	0.6667	0.6250	0.5882	0.5556	0.5263	0.5000
w_1	0.00414	0.00496	0.00576	0.00653	0.00725	0.00793	0.00855	0.00913	0.00966	0.01014	0.01058
w_2	0.00406	0.00485	0.00564	0.00638	0.00705	0.00772	0.00830	0.00883	0.00931	0.00974	0.01013
% Diff.	1.82	2.18	2.10	2.27	2.78	2.61	2.95	3.27	3.59	3.95	4.28

Buckling analysis of CCCC plate

Ibearugbulem and Ezeh (2012) compared the value of buckling coefficient, K_1 obtained for CCCC plate using the approached discussed herein and compared their data with the values, K_2 obtained by Iyengar(1988) as presented on table 4.

Table 4: Buckling coefficient parameter for CCCC plate

P = a/b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
K_1	402.71	102.83	47.471	28.307	19.667	15.218	12.79	11.477	10.845	10.667
K_2	424.97	108.22	49.753	29.51	20.384	15.685	13.121	11.734	11.066	10.878
% Diff.	5.528	5.247	4.806	4.251	3.647	3.069	2.583	2.235	2.038	1.978

Vibration analysis of SCFC plate

Ebirim et al (2014) used the present approach in determining the fundamental natural frequency parameter, λ_1 of SCFC plate and compared their value with the values, λ_2 from Leissa (1973) as presented on table 5.

Table 5: Fundamental natural frequency parameter for SCFC plate

P = a/b	Present	Lessia (1973)	% diff
0.4	22.6635	22.544	0.53
0.6	22.9371	22.855	0.36
1.0	23.8605	23.460	1.17
1.5	25.8481	24.775	4.33

5. DISCUSSION

A close at the data presented on tables 3, 4 and 5 shows that the data obtained by using polynomial function in energy method are very close to those from earlier studies. It will be noticed that higher percentage differences were recorded at aspect ratio, $a/b \leq 0.4$ or aspect ratio, $b/a = 2$. It is a common knowledge in structural engineering that a plate starts behaving as a beam when the aspect ratio, $a/b < 0.5$ or aspect ratio, $b/a > 2$. For avoidance of doubt, the use of polynomial or trigonometric functions in energy approach is only valid when the aspect ratio, $a/b \geq 0.4$ or $b/a \leq 2.5$.

Beyond this limit, result from this approach is not reliable. No wonder, higher percentage differences were recorded at aspect ratio near the invalidity region. Thus, as long as the aspect ratio is within the validity region, the data obtained from this present approach is going compare very well with data from earlier studies that used energy, equilibrium or numerical approach.

6. CONCLUSION

This approach of using polynomial function in energy method is very straight forward and non rigorous. It is very easy to satisfy the kinematic and natural boundary conditions of rectangular plate of various boundary conditions. The result from the approach compares very well with earlier known good results. Thus the result from the approach is reliable. The polynomial function can be used in any energy method (Galerkin, Ritz, work principle etc) and the outcome will be reliable.

RECOMMENDATION

Based on the outcome of this paper, it is recommended that polynomial function should be used in any of the energy methods for easy analysis of plate when approximate result can be used, especially analysis for limit state design. It is also recommended for book authors to consider publishing new books where this approach will expounded.

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