LMS and RLS based Adaptive Filter Design for Different Signals

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ABSTRACT

In this paper Adaptive filter is designed and simulated using different algorithms for noise reduction in different signals. The developed filter has been analyzed using Least Mean Square (LMS), Normalized Least Mean Square (NLMS) and Recursive Least Squares (RLS) algorithms for sinusoidal, chirp and saw-tooth signals. The performance of developed filter has been compared in-terms of Rate of Convergence and Minimum Mean Square Error (MMSE). The models for all algorithms are developed and simulated using MATLAB-SIMULINK. The simulated results show that RLS algorithm based filter provides better convergence rate at the cost of degraded MMSE as compared to LMS and NLMS. It can also be observed from the results that noise cancellation is better in saw-tooth signal as compared to sinusoidal and chirp signals.

Keywords: *Noise Cancellation, Adaptive Filter, MMSE, LMS, NLMS, RLS.*

I. INTRODUCTION

A Digital communication system consists of a transmitter, channel and receiver connected together to transmit the information from transmitter to receiver. In the transmission process, noise from the surroundings automatically gets added to the signal [1]. There are many factor that can produce this noise such as interference, delay, and overlapping. Noise problems in the surrounding have received attention due to the tremendous growth of technology that has led to noisy engines, heavy machinery, high electromagnetic radiation devices and other noise sources [2].The conventional method of estimating a signal corrupted by additive noise is to pass it through a filter that suppresses the noise while leaving the signal relatively unchanged i.e. direct filtering. The designing of these kinds of filters is the optimal filtering, which can be applied when some information about the reference noise signal is available. The noise cancellation filters has many applications in the arias of speech processing, echo cancellation and enhancement, antenna array processing, biomedical signal and image processing and so on, which is originated with .the pioneering work of Wiener and was extended and enhanced by the work of Kalman, Bucy, and others [3].

Filters used for the noise cancellation can be fixed or adaptive. The design of fixed filters requires prior information of both the signal and the noise. i.e. if we know the signal and noise beforehand, we can design a filter that passes frequencies contained in the signal and rejects the frequency band occupied by the noise. On the other hand, Adaptive filters are capable to adjust their impulse response automatically, and their design requires little or no prior knowledge of signal or noise characteristics [3,4].The aim of an adaptive filter in noise cancellation is to separate the noise from a signal adaptively to improve the signal to noise ratio [4]. The circuit for noise cancellation using adaptive filter is shown in Figure 1. The Adaptive Noise Canceller (ANC) has two inputs: primary and reference input. The primary input receives a signal ‘x’ from the signal source which is corrupted by noise ‘n’ that is uncorrelated with signal, and the reference input receives a noise ‘n_0’, which is uncorrelated with signal but correlated with noise n. The n_0 is passed through the adaptive filter to produce the close estimation of input noise i.e. y(n). This estimated noise is then subtracted from the corrupted signal that produces the estimation of error e(n).

Adaptive filters have gained attention from the designers over last many years. As a result, number of algorithms has been developed which are computationally efficient [5]. The normal adaptive algorithms which are generally used to perform weight updation of an adaptive filter are: the LMS (Least Mean Square), NLMS (Normalized Least Mean Square) and the RLS (Recursive Least Square) algorithm [6].

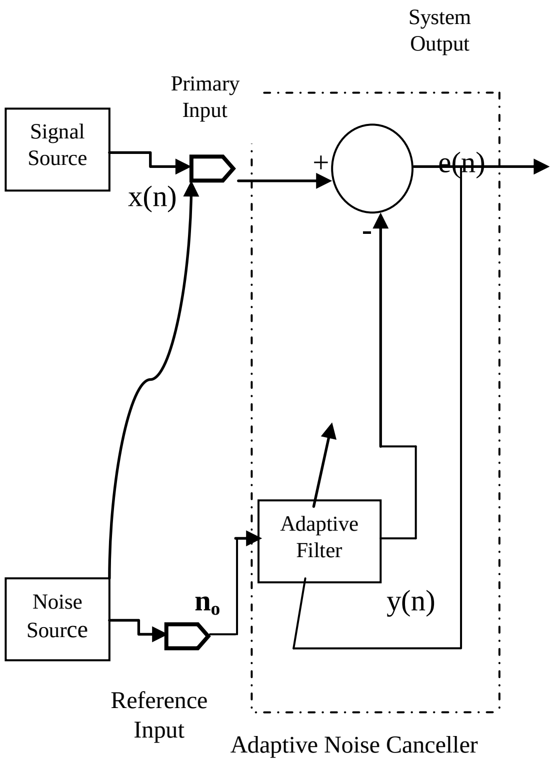

Figure1: Adaptive filter based Noise Cancellation System

II. ADAPTIVE ALGORITHMS

Least Mean Square (LMS) and Recursive Least Square (RLS) are commonly used adaptive algorithms. The LMS is one of the easiest algorithms used in the adaptive noise cancellation, because it uses the error signal to calculate the filter coefficients[3]. The output y(n) of FIR filter can be calculated from Eq. (1).

$$y(n) = \sum_{m=0}^{N-1} w(m)x(n-m) \qquad (1)$$

Where N is the order of filter and n is the number of iterations. The error signal can be calculated by Eq. (2).

$$e(n) = d(n) - y(n) \qquad (2)$$

This error signal is used to update the filter weights w(n+1) by using the current weight value w(n) as shown in Eq. (3).

$$w(n+1) = w(n) + \mu e(n)x(n) \qquad (3)$$

Here μ is the convergence factor which is used to determine the filter convergence speed as well as the overall circuit behavior. *e*(n) and *x*(n) are error vector and input vector respectively. Selection of a suitable value for μ is important to the performance of the LMS algorithm, if the value is too small, the time takes to converge by adaptive filter on the optimal solution will be too long. If μ is too large the adaptive filter becomes unstable and its output diverges. *y*(n) denotes the filter output, whose value is written in Eq. (1), *e*(n) is estimated error signal which is obtained by substituting the value of y(n) in Eq. (2).The computation of estimated error is based on currently estimated tap weight vector w(n). Right hand side of Eq.(3) is tap adjustment that is applied to current estimation of *w*(n) [5-7].

The form of algorithm describe by Eq. (1) to (3) is a complex form of LMS algorithm. At each iteration it requires knowledge of current values of *d*(n), *x*(n) and *w*(n). The iteration is started with initial value of weight vector *w*(n)=0. The LMS algorithm is a stochastic gradient algorithm that requires iteration of each tap weight in the filter in the direction of the gradient of the squared amplitude of an error signal *e*(n) with respect to that tap weight. It is nothing but an approximation of the steepest descent algorithm, which uses an instantaneous estimate of the gradient vector. The estimate of the gradient is done completely based on the basis of sample values of the tap input vector x(n)and an error signal *e*(n). LMS algorithm iterates over each tap weight w(n) in the filter, rotating itself in the direction of the approximated gradient. In LMS algorithm a big step size μ is needed, which is used to maximize the convergence speed and particularly for the stable operation this algorithm is required, also a theory which is valid beyond an infinitesimally small step size range is required.

The main drawback of the LMS algorithm is that it is very sensitive towards the scaling of its input sequence vector x(n). This makes it very hard to choose a step size parameter μ that guarantees stability of the algorithm. The normalized LMS (NLMS) is based on the principle of minimal disturbance which states from one iteration to the next the weight vector w(n) of an adaptive filter must be changed in a minimal manner, subject to a constraint imposed on the updated filters output. The Normalized least mean square (NLMS) is an extension of the LMS algorithm that solves this problem by normalizing with the power of the input. The step size [7-8] for NLMS can be calculated from the Eq. (4). From the equation it is clear that its step size is variable.

$$\mu(n) = \frac{\alpha}{c + \|x(n)\|^2} \tag{4}$$

Where α is the NLMS adaption constant, which optimize the convergence rate of the algorithm and should satisfy the condition as $0<\alpha<2$, c is the normalization constant, which is always less than 1. The filter weights using NLMS algorithm are updated by the Eq. (5).

$$w(n+1) = w(n) + \frac{\alpha}{c + \|x(n)\|^2} e(n)x(n) \tag{5}$$

The RLS algorithm is known for its excellent performance when working in time varying environments but at the cost of an increased computational complexity and it also suffer with some stability problems. In this algorithm the filter tap weight vector is updated using Eq. (6).

$$w(n) = \overline{w}^T(n-1) + k(n)\overline{e}_{n-1}(n) \tag{6}$$

Eq. (7) and Eq. (8) provides intermediate gain vector which is used to compute tap weights [8].

$$k(n) = u(n)/\lambda + x^T(n)u(n)) \tag{7}$$

$$u(n) = \overline{w_\lambda^{-1}}(n-1)x(n) \tag{8}$$

Where λ is a small positive constant tends to, but smaller than 1. The filter output is calculated using the filter tap weights vector of previous iteration and the current input vector as shown in eq. (9) and error signal can be calculated as shown by eq. (10).

$$\overline{y_{n-1}}(n) = \overline{w}^T(n-1)x(n) \tag{9}$$

$$\overline{e_{n-1}}(n) = d(n) - \overline{y_{n-1}}(n) \tag{10}$$

In the RLS algorithm the estimates of previous samples of output signal, error signal e(n) and filter weights is required that leads to higher memory requirements to store all the values.

III. ANC MODEL SIMULATIONS

Simulation based on three different types of signals mixed with random noise. Signals are Sinusoidal signal, chirp signal and saw-tooth signal. Each signal has been corrupted by random noise. Then the convergence behaviors of the LMS, NLMS and RLS algorithms for these signals have been analyzed. After corrupting the signals with noise, these were passed through the simulation of the adaptive filter, and their error recovery rate and simulation time was calculated. The analysis of the results offered useful insight into the characteristics of the algorithms. The simulink model of the propose algorithm is shown in Figure 2.

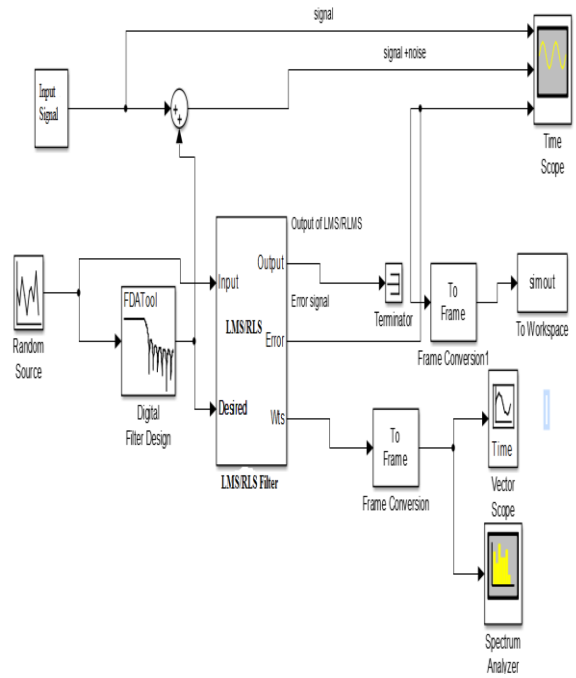

Figure 2: Simulink model for Adaptive noise Canceller

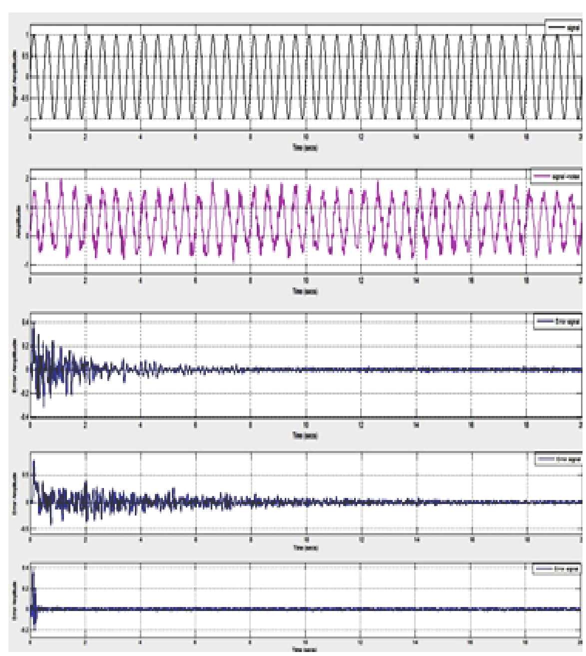

Figure 3: Adaptive Noise Cancellation for sinusoidal signal

The simulation of the adaptive algorithms i.e. LMS, NLMS and RLS for different signals is carried out with the following specifications: Filter order N=20, step size μ= 0.1 and Simulation time=20 seconds. Figure 3 shows the simulated results for sinusoidal signal for LMS, NLMS and RLS algorithms respectively.

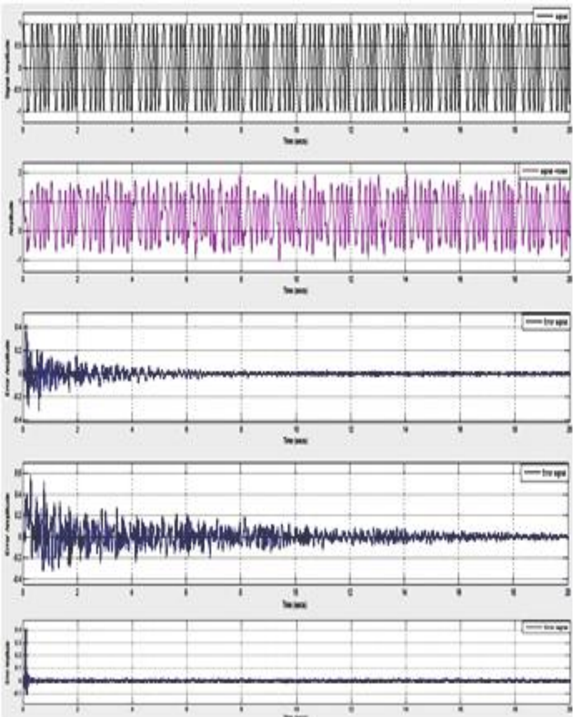

Figure 4: Adaptive Noise Cancellation for chirp signal

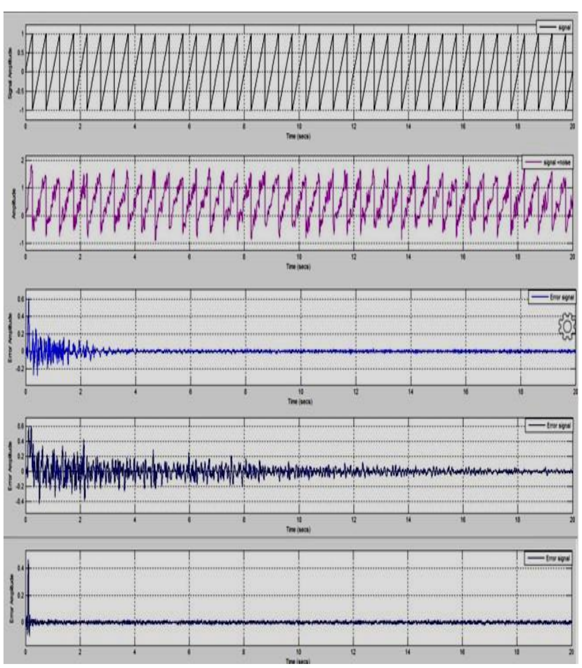

Figure 5: Adaptive Noise Cancellation for saw-tooth signal

IV. RESULT ANALYSIS

In this paper MATLAB- SIMULINK based ANC models for LMS, NLMS and RLS adaptive algorithms are compared on the basis of convergence rate and minimum mean square error (MMSE). Tabular method has been used to compare the results.

Table1 : Comparison of Covergeance Rate

Signal Type	Convergence Rate in Seconds		
	LMS	NLMS	RLS
Sinusoidal	8.2	15.25	0.75
Chirp	6.8	13.75	0.45
Saw-tooth	3.1	12.5	0.38

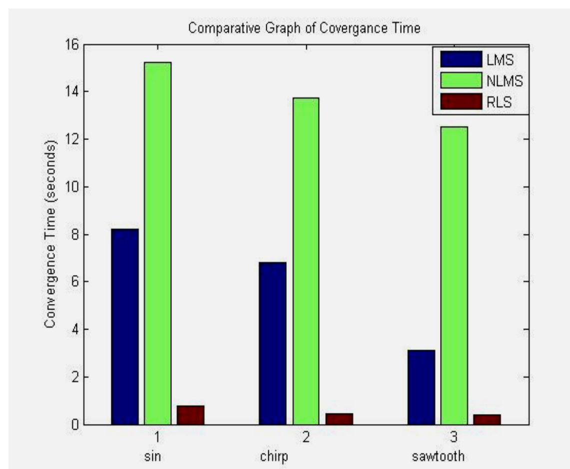

Figure 6: Convergence rate bar chart

Table 2: Mean Square Error comparison of ANC models

Signal Type	Mean Square Error		
	LMS	NLMS	RLS
Sinusoidal	1.6×10^{-2}	6.9×10^{-2}	2.9×10^{-4}
Chirp	1.6×10^{-2}	7.3×10^{-2}	2.44×10^{-4}
Saw-tooth	1.4×10^{-2}	7.0×10^{-2}	2.51×10^{-4}

V. CONCLUSION

In this paper LMS, NLMS and RLS based adaptive filters have been designed and analyzed for sinusoidal, chirp and saw-tooth signals. These algorithms are compared in-terms of convergence rate and mean square error (MSE). Simulation results show that the RLS algorithm has fastest convergence rate with degraded minimum mean square error (MMSE) as compared to LMS and NLMS. It is also clear from the simulated results that Adaptive Noise Cancellation performance for saw-tooth signal is better as compared to sinusoidal and chirp signals.

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