Group Delay Analysis of IIR Butterworth Filter for different Orders

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ABSTRACT
Generally Digital Signal Processing is Mathematical manipulation of the signals. The manipulation parameters may get differ from one technique to another. In this paper IIR Butterworth filters has been designed for various orders to analyze group delay. The developed IIR filters are designed and simulated using matlab with different filter orders. The simulated results shows that IIR filters with lower order have better group delay. Whereas IIR filters with higher filter order shows better magnitude response. This results because, as the signal passes through more processes, the distortion increases which result in non linearity and then in group delay.

KEYWORDS: *Butterworth, DSP, Group delay, IIR, MATLAB*

1. INTRODUCTION
Digital signal processing has been revolutionized the technical wardrobe of electronics. Signals generate in the natural form in the real world. Therefore signals need to be processed so that the information they contain can be displayed, analyzed or converted to another type of signals that may be of use. In the real world analog products detect signals such as sound, light, temperature or pressure and manipulate them. Before digital signal processing, the signal coming from the real world needs to be converted in to the digital form. Therefore convert these real world signals into the digital format of 0's and 1's [1]. This includes ADC at the transmitter End and DAC at the receiving End. From here the DSP takes over by capturing the digitized information. The manipulation of this digitized information under the DSP includes:-
1.Transformation (shifting, folding, time scaling) 2.Convolution (time convolution, frequency convolution) 3.Correlation (auto correlation, cross correlation). The signal is modified by using the above manipulation techniques.
In Signal Processing, the signal is processed to get the modified signal. This modification may be amplification or filteration as per the requirement of the application.

The DSP performs multiplication in a single cycle by implementing all shift and add operations in parallel. The circuitry is relatively complex and consumes a considerable number of transistors. The benefit is very fast multiplication, which is required for processing most digital signals .When general-purpose DSPs are not fast enough, the signal is either processed using analog circuits (which may have some drawbacks), or in specialized DSP hardware designed only for that task. This eliminates many of the benefits of a programmable DSP . Digital signal processing, by its nature, requires many calculations of the form:

$$A = B*C+D \qquad (1)$$

This may appear to be a simple task, but when speed is also required, we find that specialized, dedicated hardware to perform this task very easy. This may appear to be a simple task, but when speed is also required, we find that specialized, dedicated hardware to perform this task is very useful. Multiply, Accumulate (MAC). Most DSPs have a specialized instruction that allows them to multiply, add, and save the result in a single cycle. This instruction is usually called MAC (short for Multiply, Accumulate). Before digital signal processing, the signal coming from the real world needs to be converted in to the digital form. A Digital filter is a mathematical algorithm implemented in hardware or software that operates on a digital input signal to produce a digital output signal for the purpose of achieving a filtering objective [1].

2. IIR FILTER
Now our main concern is analysis of group delay at different orders for IIR Butterworth lowpass filter. Whereas Infinite impulse response (IIR) is a property applied to many linear time-invarient systems. Common examples of linear time-invariant systems are most electronic and digital filters. Systems with this property are known as IIR systems or IIR filters, and are distinguished by having an impulse response which does not become exactly zero past a certain point, but

continues indefinitely. This is in contrast to a finite impulse response in which the impulse response h(t) become exactly zero at times t>T for some finite T, thus being of finite duration [2]. The IIR filters have much better frequency response than FIR filters of the same order [3]. The input and output signals to the filter are related by the convolution sum.

$$y(n) = \sum_{k=0}^{\infty} \mathrm{h(k)x(n-k)} \qquad (2)$$

$$\mathrm{y(n)} = \sum_{k=0}^{n-1} \mathrm{h(k)x(n-k)} \qquad (3)$$

The equations (1) and (2) represents IIR and FIR filters respectively. Noting that x(n), y(n), and h(n) represents input, output and unit impulse response respectively of the filter. Where n is the order of the filter. In practice, it is not feasible to compute the output of the IIR filter using (1) because the length of its impulse response is too long , that is infinite. Instead of this the IIR filtering equation is expressed in recursive form. That is

$$y(n) = \sum_{k=0}^{n} b_k x(n-k) + \sum_{k=1}^{n} a_k y(n-k) \qquad (4)$$

Where a_k and b_k are the coefficients of the filter[4].

There are different analysis and design techniques for the IIR filters. For example: frequency analysis, phase analysis, noise analysis, order analysis etc .In this paper the main body backbone point is group delay analysis using at different orders. The actual group delay depends on the filter order (the higher the order, the higher the delay). The use of infinite impulse response (IIR) filters seems reasonable, because it is well known that IIR filters can have dramatically lower order than finite impulse response (FIR) filters with similar performance. And at lower orders, the performance of IIR filters in reference of group delay is better than at higher orders . However, if you subtract the offset in the group delay due to the filter order, the group delay of the designed filter tends to match the desired group delay [5]. A digital filter is a mathematical algorithm implemented in hardware and software that operates on a digital output signal to produce a required output. This output is further converted to the analog form for user interface. Because most of the real world signals are found in the analog form.

The filter implementation is of two types, convolution and recursive. The filters having finite impulse response are called FIR filters and the filters having which are recursive in nature having infinite impulse response are IIR filters. Due to this recursive nature their phase characteristic is non-linear. Therefore when phase is essence, IIR filters use should be avoided. On vice versa, if phase is not important in realization, then IIR filters are best choice to use. A sufficiently large delay variation can cause problems such as poor fidelity in audio or it may be inter symbol interference (ISI) in the demodulation of digital information from an analog carrier signal. High speed modems use adaptive equalizers to compensate for non-constant group delay [6]. Digital filters with linear phase responses, that is, constant group delay responses are needed in many applications for signal processing, image processing, waveform transmission and so on [7]. The specific application of IIR filters is optimizing a set of feedback coefficient to minimize the desired cost function [8].

3. IIR BUTTERWORTH LOWPASS FILTER DESIGN SIMULATIONS

The group delay occurs due to non-linearity of the signals. This non-linearity introduced during the processing of the signal. This paper deals with group delay analysis at different orders, along with defining some certain parameters which are assumed constant like sampling frequency Fs=48KHz, cut-off frequency=10.8KHz and Attenuation= 3db. This analysis has been done by using the butterworth lowpass filter by matlab coding. Firstly the above values are defined and then changing the value of order, the result has been simulated. This simulated result gives the relation between order and group delay, second is order and magnitude.

Magnitude response of Butterworth lowpass filter at N=10 is shown in the figure 1, Where N is the order of the filter. As the order increase, the magnitude response gives the better results.

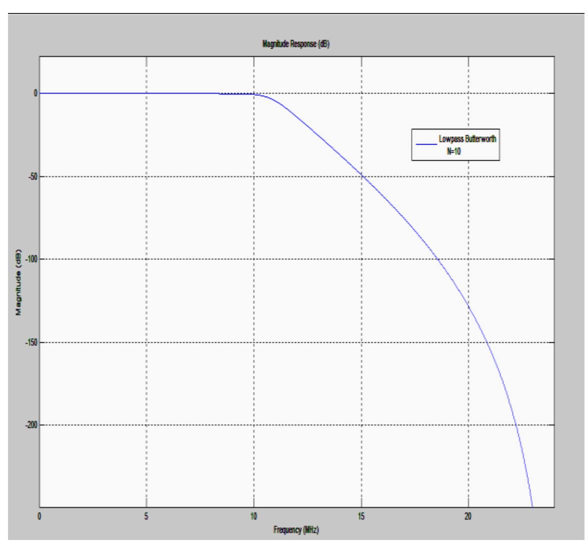

Figure 1 Magnitude response at N=10

Magnitude response at N=20, which is better than the N=10 is shown in the figure 2, where N is the order of the filter. The transition slope explains the performance of the IIR filter in terms of magnitude response. Higher the order results in lower transition slope of the waveform, from passband to stopband if there is large change in frequency then corresponding change in magnitude is small.

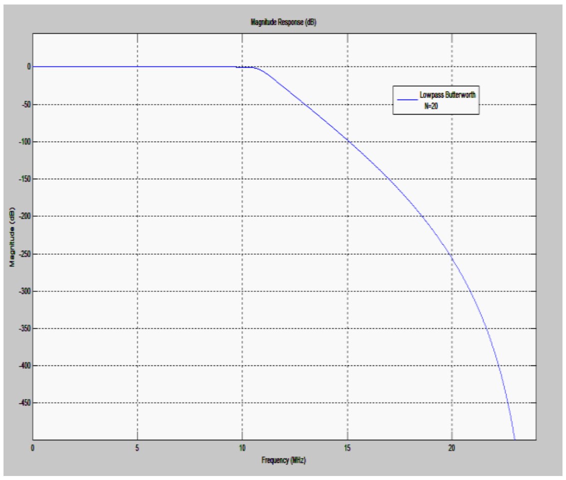

Figure 2 Magnitude response at N=20

Figure 3 shows the Group Delay response of Butterworth lowpass filter at N=10, Where N is the order of the filter. As the order increases, the group delay gives the poor results.

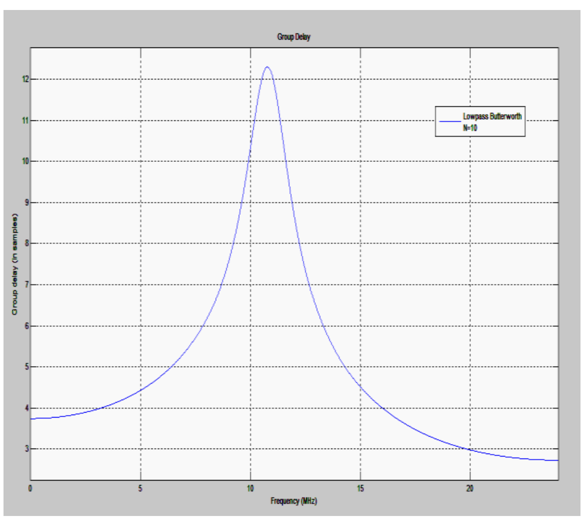

Figure 3 Group delay response at N=10

Group delay response at N=20, which is poor than N=10, corresponding to the peak sharpness of the resulting waveform. The group delay sharpness increases as the order of the filter increases. As we can see from the fig 2 & fig 4, at N=10, group delay peak represents itself at 12, as the order changes to 20 its sharpness reaches at 27. As the order increases, sharpness increases.

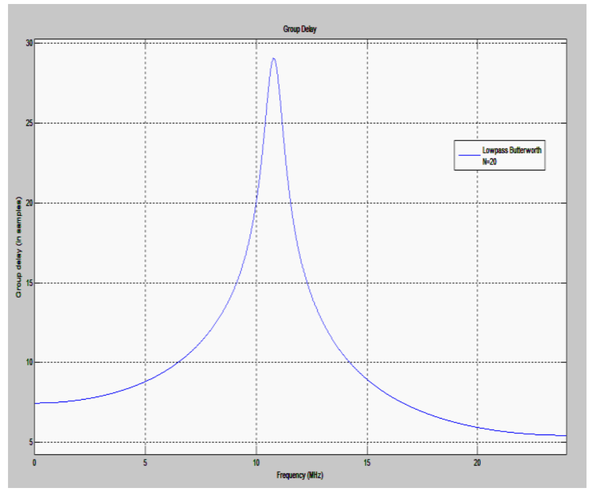

Figure 4 Group delay response at N=20

When the order of the filter in design specification is taken as N=30. Again on increasing the order of the filter it gives better performance.This performance parameter is on the basis of the observation of transition width, from the passband to the stopband. Higher the order, better corresponding results of IIR filters in magnitude.

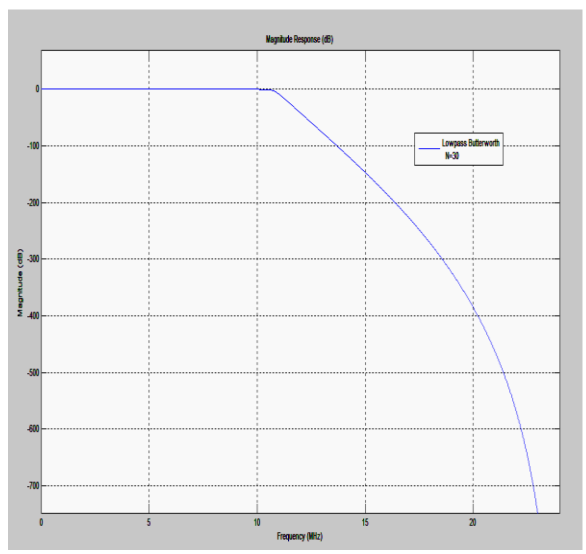

Figure 5 Magnitude response at N=30

The magnitude response at N=40, which is again better than the lower order response. This result has been shown in the figure 6.

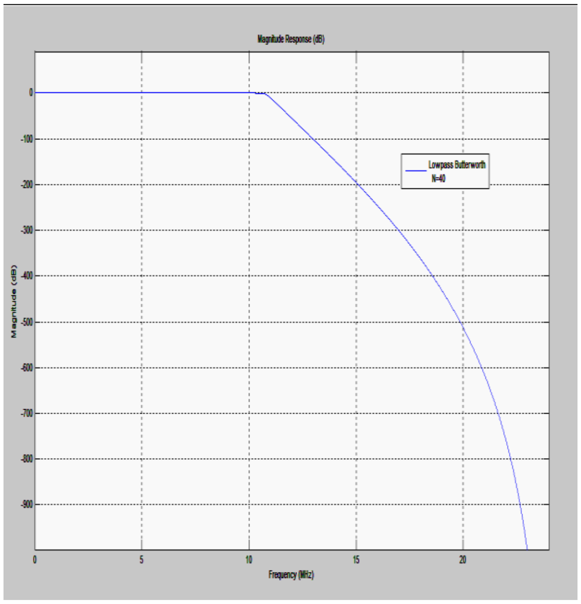

Figure 6 Magnitude response at N=40

Figure 6 shows the group delay response at N=30, In this case as the order increases the result becomes worsen than the before one.

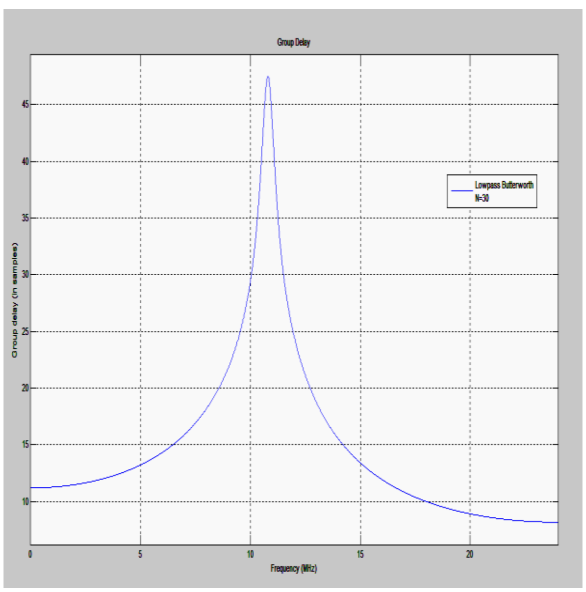

Figure 7 Group Delay response at N=30

The degrading performance of the group delay is shown in the figure 8, at N=40. This is more worst than the before one as at N=30. The low flatness of the waveform shows the worst result for group delay which is very high.

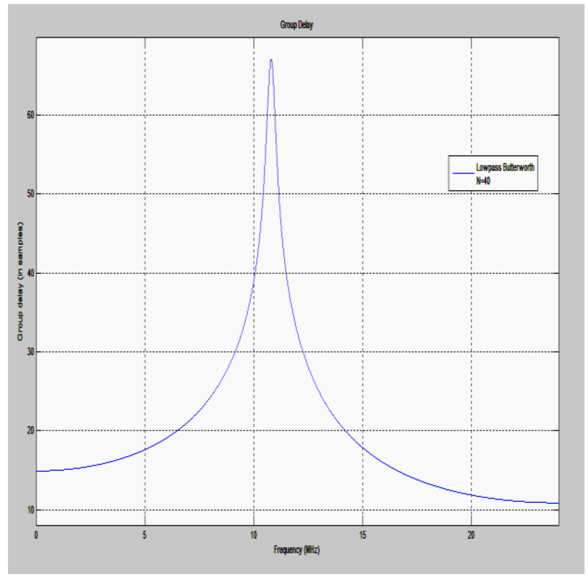

Figure 8 Group delay response at N=40

4. COMPARATIVE ANALYSIS

In the figure 9, there are different wave shapes at different orders. The evaluation from the figure is like, when the filter order is 10, the group delay sharpness is less. As the order increases from 10 to 20 the sharpness increases. Again when the filter order increases from 20 to 30 & 30 to 40 the respective sharpness of the waveform increases, which results in poor performance.

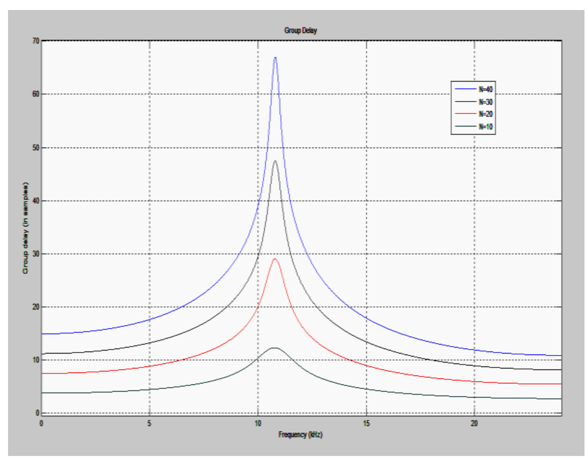

Figure 9 Group delay response at different orders.

Similarly, in figure 10 also, when the filter order increases from 10 to 20 it results in better magnitude response. Again, when the order increases from 20 to 30 & from 30 to 40, gives better magnitude response. Therefore from both the figures, we can conclude that, the group delay and magnitude response are inversely proportional to each other in their performance.

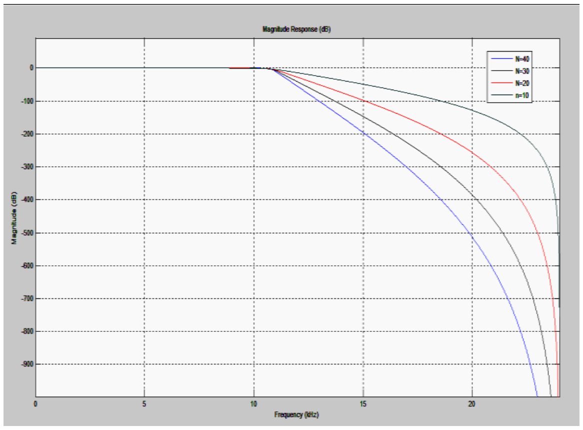

Figure 10 Magnitude responses at different orders.

5. CONCLUSION

The proposed method provides an easy and less complex method for analysis of group delay at different orders. In the IIR Butterworth filter as the order increases the sharpness of the group delay waveform also increases respectively .Which concludes that higher the order, results will be poor in reference of group delay. Vice versa if we consider the magnitude response on the above mentioned orders, then the IIR Butterworth gives the better performance at higher orders because the size of transition width get reduced but the cost factor increases due to the increase in the number of multipliers and adders.

ACKNOWLEDGEMENT

The authors would also like to thank Director, National Institute of Technical Teacher's Training & Research, Chandigarh. HOD, Electronics and Communication Engineering department, Govt Polytechnic Jhajjar, Haryana, India for their constant inspirations and support throughout this research work.

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