Analysis of Power Spectrum Estimation Using Welch Method for Various Window Techniques

Pranay Kumar Rahi[1], Rajesh Mehra[2]
[1]ME Scholar, [2]Associate Professor,
[1,2]Department of Electronics & Communication Engineering
National Institute of Technical Teachers' Training & Research
Chandigarh, UT, India

ABSTRACT

In this paper, Power Spectral Estimation (PSE) scheme for variable data length using Rectangular, Blackman, Hanning, Bartlett and Hamming window with Welch Method. Welch method includes the periodogram having advantage of possible implementation using the Fast Fourier Transform (FFT). This method gives good resolution when optimal selection of data length samples is done. The PSE based on both Rectangular as well as Hamming window has been designed and simulated using MATLAB. It can be observed that the Rectangular and Hamming give better results than other windows like Bartlett, Hanning and Blackman window.

Keywords- *DSP, FFT, FIR, MAC, PSE, PSD, Welch Method*

1. INTRODUCTION

Power Spectrum estimation (PSE) is one of the most important areas of research and applications in Digital Signal Processing (DSP). PSE techniques and algorithm that nonparametric or classical methods based on the periodogram. PSE is the spectral characteristics of signals characterized as random processes. The autocorrelation function of a random signal is the appropriate statistical average that will use for the characterizing random signals in the time domain, and the power density spectral is the Fourier transform of the autocorrelation function, provides the transformation from the time domain to the frequency domain [1].

A power spectrum describes the energy distribution of a time series in the frequency domain. Energy is a real-valued quantity, so the power spectrum does not contain phase information. Because a time series may contain non-periodic or asynchronously-sampled periodic signal components, the power spectrum of a time series typically is considered to be a continuous function of frequency.

The nonparametric Welch method in which the power of any input is guesstimation at different frequencies. It is an improvement on the Periodogram (a method of estimating the autocorrelation of finite length of a signal) spectrum estimation method where signal to noise ratio is high and reduces noise in the estimated power spectra in exchange for reducing the frequency resolution. Periodogram is easy to compute and have limited ability to produce accurate power spectrum estimation. It is biased when dealing with finite windows. As data length increase, the rate of fluctuation in this is also increase [2]. An improved version of the periodogram is Welch's method. A more modern nonparametric technique is the multitaper method (MTM).

The goal of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data. Estimation of power spectra is useful in a variety of applications, including the detection of signals buried in wide-band noise. Spectrum analysis has found use in the study of communication engineering signals, event-related or stimulated responses of the human electroencephalogram (EEG) in the diagnosis of brain diseases, other biological signals, meteorological industrial process control and the measurement of noise spectra for optimal linear filters [3].

The deleterious effects of spectral leakage and smearing may be minimized by windowing the data by a suitable window function. The sampled data value are multiplied point to point by the sampled value of selected window function. The equivalent noise bandwidth, processing gain, worst-case processing loss, and minimum resolution are considered for choosing a suitable window. In overlap correlation averaging the spectrum of the windowed data directly leads to a significantly improved estimate of the spectrum [4].

2. POWER SPECTRUM ESTIMATION

PSE is most important application area in Digital Signal Processing. There are mainly two types of power spectrum estimation (PSE) method: Parametric and nonparametric. Parametric or non-classical methods an analyzed process is replace by an appropriate model with known spectrum. Nonparametric or classical methods do not make any assumption on the data generating process. **Parametric methods** are based on parametric models of a time series, such as AR models, moving average (MA) models, and autoregressive-moving average (ARMA) models. Therefore, parametric methods also are known as model-based methods. To estimate the PSD of a time series with parametric methods, you need to obtain the model parameters of the time series first.

In **Nonparametric methods**, which include the periodogram method, Welch method and the blackman-Turkey methods, are based on the discrete Fourier transform. In these methods no need to obtain the parameters of the time series. All these methods have the advantage of possible implementation using the FFT, but with the disadvantage in the case of short data lengths of limited frequency resolution. Also, considerable care has to be exercised to obtain meaningful result. Parametric methods on the other hand can provide high resolution in addition to being computationally efficient. The most common parametric approach is to derive the spectrum from the parameters of an autoregressive model of the signal [4]. In Bartlett method, divide the signal into blocks, find their periodograms and average to get the Power spectrum. (The data segments are non-overlapping). The final effect is true power spectrum convolved with a window. Due to windowing (leakage frequency due to side lobes) the frequency resolution is low.

In Welch Method, data segments can be overlapping and Window the data (signal) before computing Periodogram (we may use different windows for each segment). This method has better precision but less frequency resolution than Bartlett method. In Blackman-Tukey Method, windowed the auto-correlation sequence and take Fourier transform to get power spectrum estimate (Periodogram) in effect we smooth out the Periodogram. It has better variance (even at large lags) and better precision than above two methods, but frequency resolution is less than the others [5].

The estimates are based entirely on a finite record of data, the frequency resolution is equal to the spectral width of rectangular window of length N, which is approximately 1/N at the -3dB points. The estimation techniques decrease the frequency resolution in order the variance in the spectral estimate. The estimates are computed at discrete frequencies [1].

3. WELCH METHOD

The Welch method [Welch 1967] is an improved estimator obtained by chaning the Bartlett method in two respects. First, the data segments in the Welch method are allowed to overlap. Second, each data segment is windowed prior to computing the periodogram.

The method consists of dividing the time series data into (possibly overlapping) segments, computing a modified periodogram of each segment, and then averaging the PSD estimates. The result is Welch's PSD estimate. Welch's method is implemented in the Signal Processing Toolbox function. By default, the data is divided into eight segments with 50% overlap between them. A Hamming window is used to compute the modified periodogram of each segment.

The averaging of modified periodograms tends to decrease the variance of the estimate relative to a single periodogram estimate of the entire data record. Although overlap between segments tends to introduce redundant information, this effect is diminished by the use of a nonrectangular window, which reduces the importance or weight given to the end samples of segments (the samples that overlap).

In the Welch method L data sections of length M are overlapped and the periodograms are computed from the L windowed data sections. The periodograms are normalized by the factor U to compensate for the loss of signal energy owing the windowing procedure.

$$U = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n) \qquad (1)$$

The Welch power density spectral estimate, $P_{WE}(f)$, is

$$P_{WE} = \frac{1}{L}\sum_{j=0}^{L-1} P_j(f) \qquad (2)$$

The expected value of the Welch estimate is

$$E[P_{WE}(f)] = \frac{1}{L}\sum_{j=0}^{L-1} E[P_j(f)] = E[P_j(f)] \qquad (3)$$

For 50% overlap (L=2K)

$$var[P_{WE}(f)] \approx \left(\frac{9}{8L}\right) P^2(f) \qquad (4)$$

Which is less than for the Bartlett by the factor 9/16=0.56 [4].

4. RESULT

In this paper, we are trying to show the Data length effect on resolution with data sequence of 312 samples, fs = 1000 and Overlay 50%.

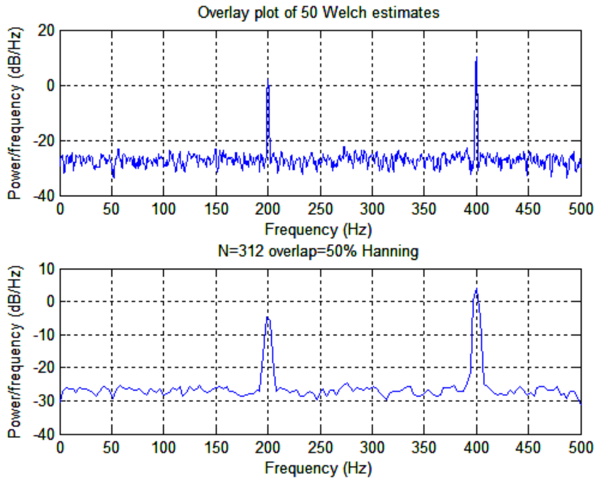

Fig. 1. Data sequence in Hanning window

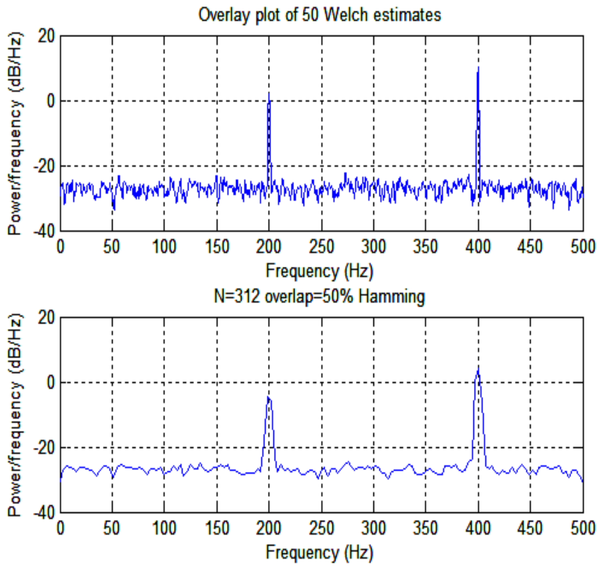

Fig. 2. Data sequence in Hamming window

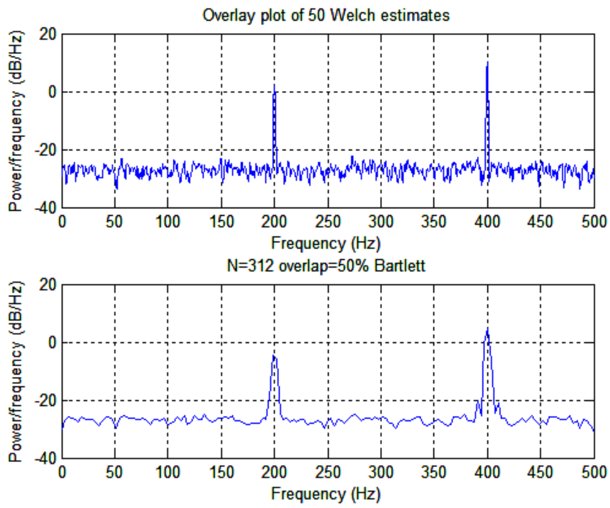

Fig. 3. Data sequence in Bartlett window

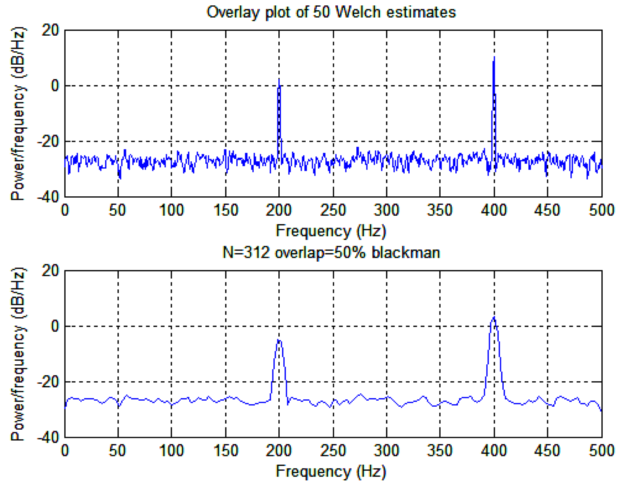

Fig. 4. Data sequence in Blackman window

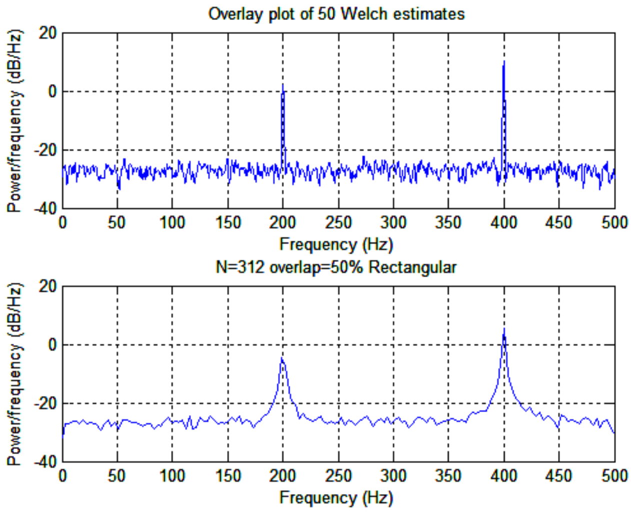

Fig. 5. Data sequence in Rectangular window

5. CONCLUSION

The Welch method provides the result on power spectrum estimation for various windows in window method. This method is used to calculate the PSD of a signal with reducing the effect of noise in DSP. In this paper the window techniques like Hamming, Hanning, Bartlett, Blackman and Rectangular window are used to extract the unwanted noise from the signal. The sampling frequency 1000Hz fixed and the number of sample are 312 with 50% overlay. We can see the differences / variations in the different windows. The purposed algorithms operate in frequency domain, where the calculation of samples is done from the frequency domain using sine waveforms. The quality of the estimate increase as the length N of the data increase, which means that the consistence. When data length is short Blackman tuckey method is better than Welch method but as the data length increase Welch Method gives excellent results. The rectangular and

Bartlett window have the clear peak in the graph showing the power spectrum estimation.

REFERENCES

[1] J.G.Proakis and D.G.Manolakis, "Digital Signal Processing Principles, Algorithms and Applications" Forth Edition, Prentice-Hall, pp.908-913, 2009.

[2] Hansa Rani Gupta, Sushila Batan and Rajesh Mehara, "Power Spectrum Estimation using Welch Method for various Window Techniques", International Journal of Scientific Research Engineering & Technology (IJSRET), Vol. 2, Issue 6, pp 389-392, September 2013.

[3] Sanjit k Mitra, " Digital signal Processing", Tata Mcgraw Hill, Second Edition, pp. 523-524, 2006

[4] Emmanual C. Ifeachor, Barrie W. Jervis,"Digital Signal Processing A practical Approach, Person Education, Second Edition, pp. 864-868, 2005.

[5] http://www.oneirix.com/www.udayankanade.org //adsp/jan2004/lecture017.pdf

[6] Mathworks, "Users Guide Filter Design Toolbox 4", March-2007.

AUTHORS

Pranay Kumar Rahi received the Bachelors of Technology degree in Electronics and Telecommunication Engi- neering from Government Engineering College, Guru Gasidas University, Bilaspur, Chhattisgarh, India in 2004, and pursuing Masters of Engineering in Electronics and Communication Engineering from National Institute of Technical Teacher's Training & Research, Punjab University, Chandigarh, India.

Rajesh Mehra received the Bachelors of Technology degree in Electronics and Communication Engineering from National Institute of Technology, Jalandhar, India in 1994, and the Masters of Engineering degree in Electronics and Communication Engineering from National Institute of Technical Teacher's Training & Research, Punjab University, Chandigarh, India in 2008. He is pursuing Doctor of Philosophy degree in Electronics and Communication Engineering from National Institute of Technical Teacher's Training & Research, Punjab University, Chandigarh, India. He is an Associate Professor with the Department of Electronics & Communication Engineering,, National Institute of Technical Teacher's Training & Research, Ministry of Human Resource Development, Chandigarh, India. His current research and teaching interests are in Signal and Communications Processing, Very Large Scale Integration Design. He has authored more than 175 research publications including more than 100 in Journals. Mr. Mehra is member of IEEE and ISTE.